PAPER

Dependence of Elastic Modulus on Inner Pressure of Tube Wall Estimated from Measured Pulse Wave Velocity

Masahiko TAKANO†, *Nonmember*, **Hiroshi KANAI**†, **Nozomu HOSHIMIYA**††, *and* **Noriyoshi CHUBACHI**†*, *Members*

SUMMARY We have proposed a non-invasive method for diagnosis of the early stage of atherosclerosis, namely, the detection of small vibrations on the aortic wall near the heart by using ultrasound diagnostic equipment. It is, however, necessary to confirm the effectiveness of such measurement of the pulse wave velocity for quantitative evaluation of the local characteristics of atherosclerosis. It is well known that Young's modulus of a tube wall, estimated from measured pulse wave velocity, depends on inner pressure because of the non-linear relationship between the inner pressure and the change of volume in the tube. The inner pressure, however, changes during the period of one heartbeat. In this experimental study, we found for the first time that Young's modulus of the tube wall, estimated from the measured pulse wave velocity, depends not only on the diastolic pressure but also on the pulse pressure and the pressure gradient of the systolic period.

key words: *atherosclerosis, pulse wave velocity, elastic modulus, inner pressure*

1. Introduction

We previously developed a noninvasive method to diagnose disorders of the cardiovascular system using ultrasound [1]. In the diagnosis of atherosclerosis, a major concern is changes in arterial wall hardness with every stage of this disease; such changes are determined by measuring the small vibrations which propagate on the artery. To date, in order to obtain an index for diagnosis of atherosclerosis, many investigators have measured the pulse wave velocity [2]. From the measured pulse wave velocity, Moens-Korteweg's equation [3] has been frequently applied to quantitative evaluation of the elastic modulus of the arterial wall. As is well known, the pulse wave is the pressure wave propagating in the cardiovascular system generated by beating of the heart. The propagation velocity c_0 of the pulse wave is given by

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†The authors are with the Department of Electrical Engineering, Graduate School of Engineering, Tohoku University, Sendai-shi, 980–8579 Japan.
††The author is with the Department of Electronic Engineering, Graduate School of Engineering, Tohoku University, Sendai-shi, 980–8579 Japan.
*Presently, Faculty of Engineering, Tohoku Gakuin University.

$$c_0 = \sqrt{\frac{Eh}{2\rho r}}, \tag{1}$$

where E is Young's modulus of the arterial wall, ρ is the mass density of inner fluid, r is the inner radius of the artery, and h is the thickness of the arterial wall. Equation (1) implies that the pulse wave velocity c_0 is proportional to the square root of Young's modulus E of the arterial wall. Hence, the stiffness of the arterial wall is quantitatively estimated by measuring the pulse wave velocity c_0. It is, however, necessary to make corrections since the pulse wave velocity c_0 and then the obtained Young's modulus E of the arterial wall strongly depend on the inner pressure; the non-linear stress-strain relation of the aortic wall contributes to this dependence. In addition, the inner pressure changes its value during one cardiac cycle.

Numerous studies on the pulse wave and its propagation velocity c_0 have been reported [2], [3]. The delay time of the pulse wave propagating between two measurement points, however, has been determined in the time domain and only rising time of the recorded pulse wave has been considered. It has been concluded that the pulse wave velocity c_0 depends on only the dimensions of the cross sectional area and Young's modulus E of the arterial wall and the diastolic pressure [3]. To discuss the non-linear elastic characteristic of the arterial wall, it is insufficient to consider only the rising time of the pulse wave.

In this paper, the dependence of the pulse wave velocity c_0 and obtained Young's modulus E on the inner pressure is experimentally evaluated by using a silicone rubber tube. Since the inner pressure varies during one cardiac cycle, not only the diastolic pressure but also the pulse pressure and the pressure gradient during systole are considered to be factors contributing to the pulse wave velocity c_0 and Young's modulus E.

2. Methods

2.1 Measurement of Incremental Elastic Modulus H for the Arterial Wall

In general, an arterial wall, which has incompressibil-

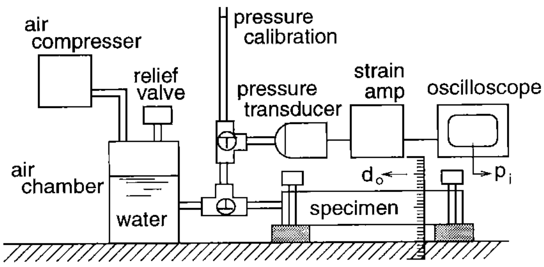

Fig. 1 Diagram showing testing of the relationship between the inner pressure p_i and the outer diameter d_o.

ity and a non-linear relationship between the stress and the strain with large amplitude, is an anisotropic viscoelastic medium. It is difficult to accurately describe the viscoelastic property of the arterial wall. Thus, the quantitative estimation of the elastic modulus of the arterial wall is of interest.

In our experiment, a silicone rubber tube is employed as a specimen. Its inner diameter and outer diameter are $d_i = 11.8$ mm and $d_o = 16.1$ mm, respectively, and its length is $l = 243$ mm. Let us assume that the tube wall is an incompressible, uniform, isotropic, cylindrical elastic shell. In the case that tube length is constant (i.e. axial strain is negligible) by clamping both ends of tube, to quantitatively evaluate the stiffness of the tube wall, the following incremental elastic modulus $H(p_i)$ [4] is employed:

$$H(p_i) = 2\left(\frac{\Delta p_i}{\Delta d_o}\frac{d_o d_i^2}{d_o^2 - d_i^2} + \frac{p_i d_o^2}{d_o^2 - d_i^2}\right), \tag{2}$$

where p_i is the inner pressure, d_i and d_o are the inner diameter and the outer diameter of the artery, respectively, and $\Delta p_i/\Delta d_o$ is the gradient of the pressure-diameter curve. The specimen has non-linearity on the stress-strain relationship. Hence, the value of $H(p_i)$, including the factor $\Delta p_i/\Delta d_o$, depends on the inner pressure p_i.

To obtain the incremental elastic modulus $H(p_i)$ by using Eq. (2), the outer diameter d_o must be measured for various values of the inner pressure p_i. Figure 1 shows a schematic diagram for testing the relationship between the inner pressure p_i and the outer diameter d_o. Since it is assumed that the specimen has incompressibility, the inner diameter d_i is calculated from the measured outer diameter d_o by

$$d_i = \sqrt{d_o^2 - d_{o0}^2 + d_{i0}^2}, \tag{3}$$

where the subscript 0 indicates that the value is obtained when no inner pressure is applied ($p_i = 0$).

2.2 Measurement of Pulse Wave Velocity c_0

We measure how the pulse wave velocity c_0 and the obtained Young's modulus E vary when the inner pressure $p_i(t)$ is changed. Figure 2 shows a pulse wave measurement system [5]. In our measurement, signals $a_A(t)$ and $a_B(t)$ of the wall vibration generated by the pulse wave are simultaneously measured using two acceleration pickups which are attached to two adjacent points A and B on the wall surface of the specimen. From the resultant signals, the delay time τ_{AB} of the pulse wave which propagates from point A to B is determined in the frequency domain by a computer. We concentrate on expansion of the tube wall in the systolic period, caused by an increase in the inner pressure $p_i(t)$. The time interval during the rise in pressure from the diastolic pressure to its peak is about 30 ms, and the Hamming window with 30 ms in time length is multiplied on the measured acceleration signals. The complex transfer function $H_{AB}(f)$ from $a_A(t)$ to $a_B(t)$ and the magnitude-squared coherence function $|\gamma_{AB}(f)|^2$ between $a_A(t)$ and $a_B(t)$ are obtained. When the pulse wave is non-dispersive, the phase $\angle H_{AB}(f)$ of the transfer function varies linearly against the frequency f. Thus, the delay time τ_{AB} is obtained from the gradient $d\angle H_{AB}(f)/df$ of the phase of the transfer function, and the pulse wave velocity c_0 is calculated by dividing the distance d_{AB} between the two points by the resultant delay time τ_{AB}. The pulse wave velocity c_0 is measured for various values of the diastolic pressure $p_d = \min_t p_i(t)$, the pulse pressure $p_p = \max_t p_i(t) - \min_t p_i(t)$, and the pressure gradient $\partial p_i(t)/\partial t$ at systole.

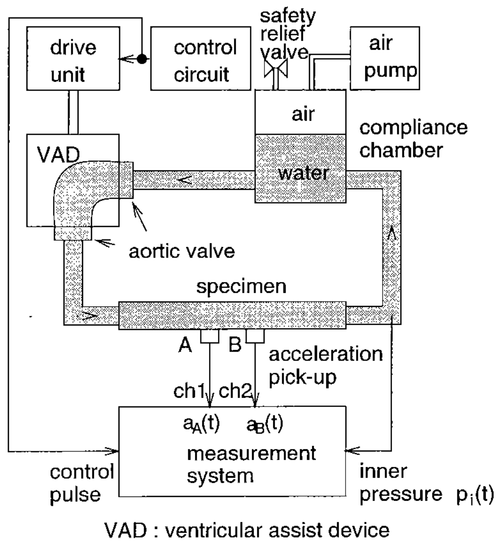

Fig. 2 A system for measurement of the pulse wave velocity using a ventricular assist device.

3. Experimental Results

3.1 Incremental Elastic Modulus H

By changing the inner pressure p_i, the outer diameter d_o is measured at each pressure by using the system shown

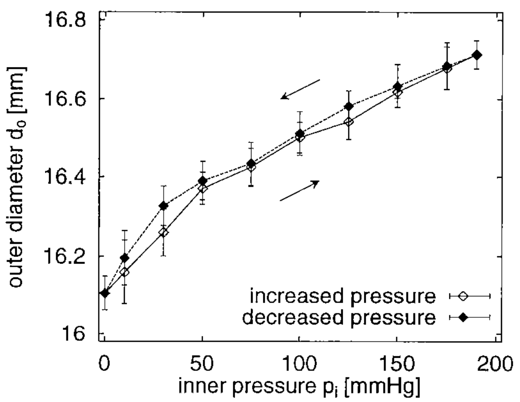

Fig. 3 Hysteretic elasticity of the silicone rubber tube.

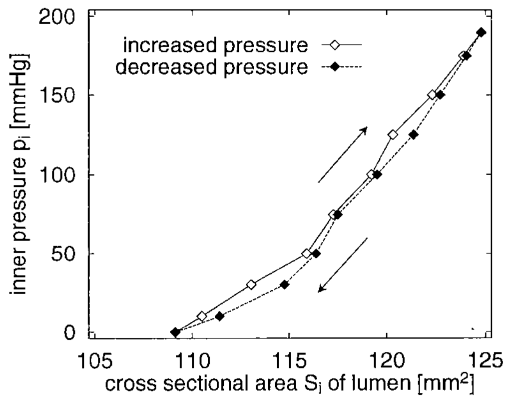

Fig. 4 Relationship between the inner pressure p_i and the cross sectional area S_i of the lumen.

in Fig. 1. The constant length condition is satisfied by clamping the specimen at both ends. The obtained relation between the inner pressure p_i and the outer diameter d_o is shown in Fig. 3. In this figure, each plot point shows the mean value and the error bar shows the standard deviation of 5-time measurements. The measurements have been done by using digital micrometer (SONY μ-mate M-30) and the ratio of standard deviation to the mean value in 5-time measurements is less than 1%.

Figure 4 shows the hysteretic elasticity of the specimen, that is, the relationship between the inner pressure p_i on the vertical axis and the cross sectional area S_i of the lumen on the horizontal axis. By assuming incompressibility of the specimen, S_i is given by

$$S_i = \frac{\pi d_i^2}{4}, \tag{4}$$

where the inner diameter d_i is calculated by Eq. (3). From Fig. 3, it is found that the gradient $\Delta p_i/\Delta d_o$ of the inner pressure varies as the cross sectional area S_i is changed.

Figure 5 shows the incremental elastic modulus $H(p_i)$, obtained from the gradient $\Delta p_i/\Delta d_o$ of Fig. 3. It is found that the incremental elastic modulus $H(p_i)$ of

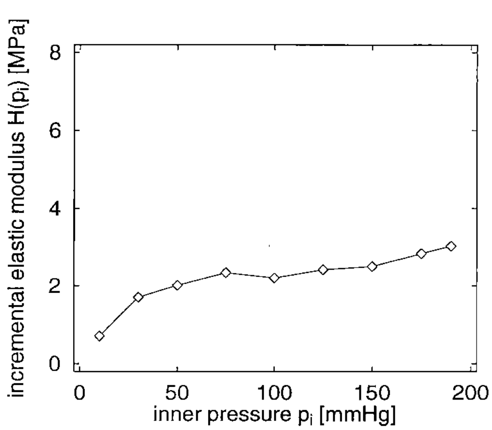

Fig. 5 Dependence of the incremental elastic modulus $H(p_i)$ on the inner pressure p_i.

the silicone rubber tube gradually increases as the inner pressure p_i is increased. Thus, the incremental elastic modulus $H(p_i)$ of the specimen markedly depends on the inner pressure p_i.

3.2 Pulse Wave Velocity c_0

For the same silicone rubber tube as in the previous measurements, the pulse wave velocity c_0 is measured to evaluate its dependence on the inner pressure $p_i(t)$. The distance d_{AB} between the two points A and B is 46.8 mm, and the sampling frequency is 5 kHz.

Figure 6 shows the results obtained by applying spectrum analysis to the resultant acceleration signals $a_A(t)$ and $a_B(t)$ when the diastolic pressure p_d is 50 mmHg and the pulse pressure p_p is 100 mmHg. Figure 6 (1) shows the resultant acceleration signals $a_A(t)$ and $a_B(t)$. Figure 6 (2) shows that the power spectra $P_A(f)$ and $P_B(f)$ of $a_A(t)$ and $a_B(t)$. Figure 6 (3) shows the magnitude-squared coherence function $|\gamma_{AB}(f)|^2$. Figure 6 (4) shows the squared magnitude $|H_{AB}(f)|^2$ of the transfer function. Figure 6 (5) shows the phase $\angle H_{AB}(f)$ of the transfer function. It is confirmed that the transfer system of the local area between points A and B is linear because the magnitude-squared coherence function $|\gamma_{AB}(f)|^2$ is almost 1 in the frequency range from d.c. to 70 Hz. From the gradient $d\angle H_{AB}(f)/df$ of the phase of the transfer function in this frequency range (broken line in Fig. 6 (5), which is obtained by using the least mean square method), the delay time τ_{AB} between two points A and B is obtained by

$$\tau_{AB} = -\frac{1}{2\pi}\frac{d}{df}\angle H_{AB}(f). \tag{5}$$

Thus, the obtained pulse wave velocity is $c_0 = 22.4$ m/s.

The dependence of the pulse wave velocity c_0 on the diastolic pressure p_d is shown in Fig. 7. It is found that the pulse wave velocity c_0 gradually increases as the diastolic pressure p_d is increased.

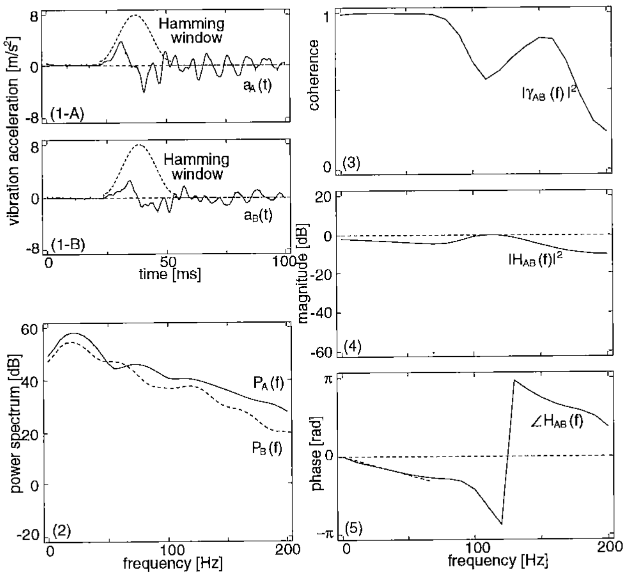

Fig. 6 The results of the spectrum analysis of the resultant acceleration signals with the diastolic pressure $p_d = 50$ mmHg and the pulse pressure $p_p = 100$ mmHg. (1) The resultant vibration signals $a_A(t)$ and $a_B(t)$. (2) The power spectra $P_A(f)$ and $P_B(f)$ of $a_A(t)$ and $a_B(t)$. (3) The magnitude-squared coherence function $|\gamma_{AB}(f)|^2$. (4) The magnitude of the transfer function $|H_{AB}(f)|^2$ from measured point A to B. (5) Phase $\angle H_{AB}(f)$ of the transfer function.

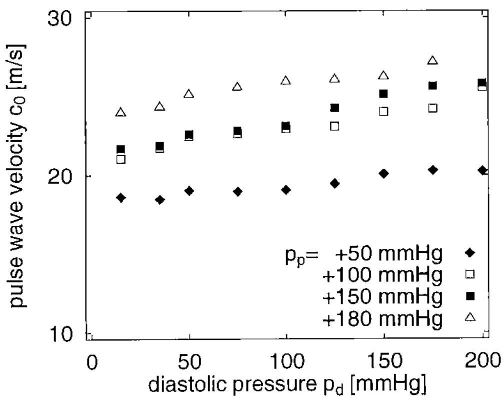

Fig. 7 Dependence of the pulse wave velocity c_0 on the diastolic pressure p_d.

3.3 Dependence of Elastic Moduli $H(p_i)$ and E on Inner Pressure $p_i(t)$

The obtained values of the elastic moduli $H(p_i)$ and E are summarized in Fig. 8. In Fig. 8, Young's modulus E is obtained from Eq. (1), where the inner radius r of the specimen is 6.0 mm, the thickness h of the tube wall is 2.0 mm, and the mass density ρ of the inner fluid is 1.0×10^3 kg/m^3. It is clear that Young's modulus E gradually increases as the diastolic pressure p_d and the pulse pressure p_p are increased.

Figure 9 shows that the dependence of Young's modulus E on the pulse pressure p_p for various values of the diastolic pressure p_d. Moreover, the dependence of

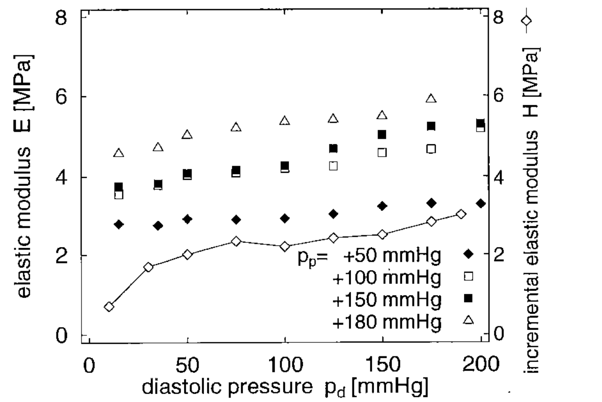

Fig. 8 Dependence of the elastic moduli $H(p_i)$ and E on the diastolic pressure p_d and the pulse pressure p_p.

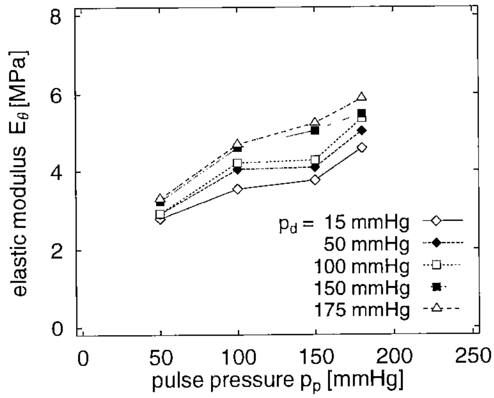

Fig. 9 Dependence of Young's modulus E on the pulse pressure p_p and the diastolic pressure p_d.

Young's modulus E on the diastolic pressure p_d slightly increases with the pulse pressure p_p.

Furthermore, the obtained Young's modulus E is larger than the resultant incremental elastic modulus $H(p_i)$ for the diastolic pressure p_d ranging from 15 to 200 mmHg. This relationship between E and $H(p_i)$ is naturally explained by the following. The dynamic elastic modulus is always larger than the static one. The incremental elastic modulus $H(p_i)$ is measured for statically applied stress; on the other hand, Young's modulus E is measured for dynamically applied stress. Thus, it is a natural consequence that Young's modulus E is larger than the incremental elastic modulus $H(p_i)$.

4. Discussions

With reference to Fig. 8, when the diastolic pressure p_d is 100 mmHg and the pulse pressure p_p is 50 mmHg, Young's modulus E is equal to 2.91 MPa; when p_d is 50 mmHg and p_p is 100 mmHg, however, E is equal to 4.04 MPa. Our major concern is what causes this difference. It can be explained as being the result of the waveform of the inner pressure $p_i(t)$ and the pressure gradient $\partial p_i(t)/\partial t$ at systole. Figure 10 shows the

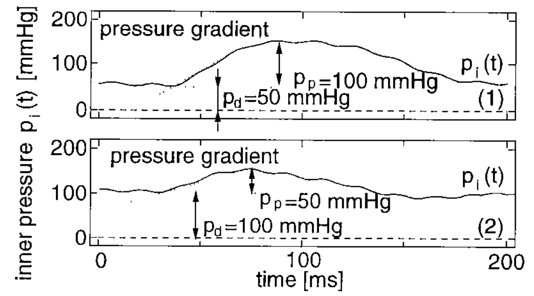

Fig. 10 Waveform of the inner pressure $p_i(t)$ and the pressure gradient $\partial p_i(t)/\partial t$ at systole. (1) The diastolic pressure p_d is 50 mmHg and the pulse pressure p_p is 100 mmHg. (2) The diastolic pressure p_d is 100 mmHg and the pulse pressure p_p is 50 mmHg.

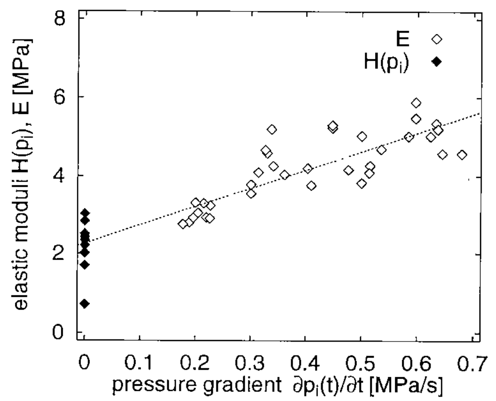

Fig. 11 Dependence of the elastic moduli $H(p_i)$ and E on the pressure gradient $\partial p_i(t)/\partial t$ at systole.

waveform of the inner pressure $p_i(t)$ and its gradient $\partial p_i(t)/\partial t$ by dashed line. For Figs. 10(1) and 10(2), the systolic pressure, which is the sum of the diastolic pressure p_d and the pulse pressure p_p is the same, although, the pulse pressure p_p is different.

From various values of the diastolic pressure p_d and the pulse pressure p_p, the pressure gradient $\partial p_i(t)/\partial t$ at systole is determined by the least square fitting to each measured waveform of the inner pressure $p_i(t)$. From Fig. 8, the Young's modulus E is also obtained at each measurement with various values of the pressure. Thus the relationship between the $\partial p_i(t)/\partial t$ and E is obtained. Figure 11 shows the dependence of Young's modulus E on the pressure gradient $\partial p_i(t)/\partial t$. In Fig. 11, the measured plot points at $\partial p_i(t)/\partial t = 0$ show the incremental elastic modulus $H(p_i)$. The broken line is obtained by the least squared fitting. Young's modulus E markedly increases as the pressure gradient $\partial p_i(t)/\partial t$ is increased at systole because the relationship between the stress and the strain on the viscoelastic medium depends not only on the strain but also on the strain velocity.

5. Conclusions

In this paper, we measured the pulse wave velocity c_0 and studied how Young's modulus E varies with various inner pressure p_i. The measurements were done with a silicone rubber tube, which has non-linearity on the stress-strain relationship. These results led to the conclusion that Young's modulus E estimated from the pulse wave velocity c_0 depends on the inner pressure p_i. The inner pressure p_i varies even in the period of one heartbeat. From our study, it was shown that not only the diastolic pressure p_d but also the pulse pressure p_p and the pressure gradient $\partial p_i(t)/\partial t$ at systole contribute to Young's modulus E obtained from the measurement of the pulse wave velocity c_0.

We used a small pressure detector and simultaneously measured the inner pressure $p_i(t)$ at the point of measurement, though in *in vivo* measurement, such an invasive method is not desirable. Consequently, when Young's modulus E obtained from the measurement of the pulse wave velocity c_0 is employed as a stiffness index of the arterial wall, it is important to correct the inner pressure $p_i(t)$ not only by the diastolic pressure p_d but also by the pulse pressure p_p and the pressure gradient $\partial p_i(t)/\partial t$ at systole.

Acknowledgements

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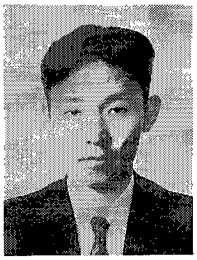
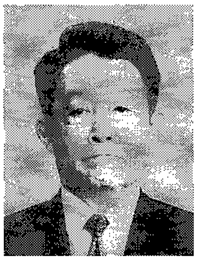


Masahiko Takano was born in Nagaoka, Japan, on April 22, 1969. He received B.E. and M.E degrees from Tohoku University, Sendai, Japan in 1993 and 1995, both in electrical engineering. Since 1995, he has been pursuing the Doctor's degree in electrical and communication engineering, Tohoku University, Japan. His current interest is in digital signal processing and ultrasonic measurement for cardiovascular disease. Mr. Takano is a member of Acoustical Society of Japan, the Japan Society of Medical Electronics and Biological Engineering, and the Institute of Electrical Engineers of Japan. He is Research Fellow of the Japan Society for the Promotion of Science.

Hiroshi Kanai was born in Matsumoto, Japan, on November 29, 1958. He received the B.E. degree from Tohoku University, Sendai, Japan in 1981, and the M.E. and the Dr. Eng. degrees, also from Tohoku University, in 1983 and 1986, both in electrical engineering. From 1986 to 1988, he was with the Education Center for Information Processing, Tohoku University, as a Research Associate. From 1990 to 1992, he was a Lecturer in the Department of Electrical Engineering, Faculty of Engineering, Tohoku University. Since 1992, he has been an Associate Professor in the Department of Electrical Engineering, Faculty of Engineering, Tohoku University. His present interest is in ultrasonic measurements and digital signal processing for diagnosis of the heart diseases and arteriosclerosis. Dr. Kanai is member of the Acoustical Society of Japan, the Japan Society of Mechanical Engineers, the Japan Society of Ultrasonics in Medicine, the Japan Society of Medical Electronics and Biological Engineering, the Institute of Electrical Engineers of Japan, the IEEE and the Japanese Circulation Society.



Nozomu Hoshimiya was born in Yokosuka, Japan, on February 8, 1941. He received the B.E., M.E., and Ph.D. degrees in electronic engineering from Tohoku University, Sendai, Japan, in 1964, 1966, and 1969, respectively. Since 1972, he has been an Associate Professor in the Department of Electronic Engineering, Tohoku University. Since June 1982, he has been a Professor at the Research Institute of Applied Electricity, Hokkaido University (Chair of Sensory Information Engineering). Since May 1988, he has been a Professor in the Department of Electronic Engineering, Tohoku University (Chair of Biomedical Electronics). His principal fields of interest are multichannel functional electrical stimulation (FES) as a neural prosthesis, especially application to the rehabilitation fields, self-organizing neural networks, especially on the recognition and generation of the spatiotemporal patterns, and physiological instrumentation. Dr. Hoshimiya has been an Ad-Com Member of the IEEE/EMBS in 1989–1990, and a member of the International Committee of the IEEE/EMBS. Since January 1994, he has been a Fellow of IEEE. He has also been an Editor-in-Chief of the Journal of the Japan Society of Medical Electronics and Biological Engineering since 1991.

Noriyoshi Chubachi was born in Kokura, Japan, on October 5, 1933. He received the B.S., M.S. and Ph.D. degrees in electrical engineering from Tohoku University, Sendai, Japan, in 1956, 1962, and 1965, respectively. In 1965 he joined the Research Institute of Electrical Communication, Tohoku University, where he was an Associate Professor from 1966 to 1978. From 1979 to 1997, he was a Professor in the Department of Electrical Engineering, Tohoku University. From 1982 to 1983, he was a Visiting Professor of Electrical and Computer Engineering, University of California, Santa Barbara. Since 1997, he has been a Professor in Faculty of Engineering, Tohoku Gakuin University, Tagajo, Japan. He has worked on ultrasonic transducers and delay lines, surface acoustic devices, acoustoelectronics, piezoelectric materials, acoustic microscopy, and related problems. Dr. Chubachi is a member of the Acoustical Society of America, the Society of Japanese Applied Physics, the Acoustical Society of Japan, the Japan Society of Ultrasonics in Medicine, the Japan Society for Nondestructive Inspection, the Japan Society of Medical Electronics and Biological Engineering, the Japan Society of Mechanical Engineers, and the Institute of Electrical Engineers of Japan. He served as Chairman, Tokyo Chapter of the IEEE Ultrasonics, Ferroelectics, and Frequency Control Society from 1987 to 1988. He is currently serving as Vice President of the Acoustical Society of Japan.